Integrated Analytical Model for Relief Logistics in Uncertain Conditions

Ardavan Babaei a, Kamran Shahanaghi b*

a Department of Industrial Engineering, Iran University of Science & Technology, Tehran, Iran
b Department of Industrial Engineering, Iran University of Science & Technology, Tehran, Iran

Abstract
The life of many people in the world is under the danger because of the accidents and unpredicted diseases, which require relief. Since, most of such cases occur in unpredictable condition, for an appropriate planning, the uncertain conditions should be investigated. Consequently, this paper investigates the integrated and multi-step process of locating, assigning and routing the air and land emergencies in the uncertain conditions in order to reach the stable solution that faces minimum changes against different conditions. In the presented model, the emergency demand is defined in the stochastic and fuzzy environment in order to show the actual conditions. The findings indicate that the models intend to create an optimum balance between cost minimization and increasing the demand satisfaction rate. However, in general conditions, the decisions should be made considering the existing conditions.

Introduction and Literature Review
Large-scale emergency events, either man-made or natural, inflict tremendous damages on human. For instance, in 2015, 341 natural disasters happened resulting in 213 million victims, 8421 deaths and billions of losses in assets (Gucha et al., 2015). In the same year, 152 man-made disasters happened which killed more than 10000 people (Swiss, 2015). In the light of such events and their impacts on human life, it is necessary for the decision makers of disaster management can to prevent casualties and destructions using scientific methods. One of the most important scientific methods for this purpose is operations research. Relief logistics is an essential part of operation research, which is used as a technique and analytical tool to provide efficient relief to affected people. It is essential to consider disaster characteristics for planning. One of the main characteristics of disasters is a large number of injured people who should be treated immediately. Therefore, the quick reaction for treating injured people has an important role in the planning of during disaster (Hashzemi et al., 2014). With regard to aforementioned needs, relief logistics can help us for good planning. A method that can help us in relief logistics is ambulance and helicopter routing and assignment problem. Response planning to meet the needs of people during natural disasters such as hurricanes, floods, and earthquakes is

* Corresponding author, Tel: 00989124210824 Email: shahanaghi@iust.ac.ir
challenging because resources of more than one type have to be delivered to the demand areas in a timely manner and in right quantities. It also requires a carefully planned process of acquiring and distributing the resources. This poses further challenge as the demand in such disasters may vary over time in terms of the type of materials (or service) or in terms of quantity. Disasters are characterized by uncertainty and unpredictability; and therefore, demand may change rapidly in such an environment (Swiss, 2015). Additionally, demand for resources in one location at a period may not exist in the next period; or, a particular location may have a very high demand in the subsequent period. The performance of emergency services is measured in terms of the response time and the total logistics cost (Shafia, et al., 2013). Therefore, if demand is not met on time, the performance of service will degrade. In order to address such a situation, a flexible and efficient emergency response system should be developed so that both social and economic losses due to the aftermath of disasters could be minimized (Balcik, et al., 2008). Therefore, location and allocation of relief distribution facilities become critical for an effective emergency response planning besides ambulance and helicopter routing and assignment problem.

Iran is a western Asian country, located in the central Asian and Caucasus region, with an area of approximately 1,650,000 km² and a population of more than 75 million people. According to the World Health Organization (WHO), cardiovascular diseases are the most prevalent causes of mortality in Iran (Bahadori et al., 2010). Unintentional accidents are the second cause of mortality. Iran with 28000 annual deaths has the highest mortality rate from road traffic accidents in the world. The road accidents rate in Iran is 20 times higher than the world average. According to the Iranian data, one person loses his/her life every 19 minutes due to car accidents (Bahadori et al., 2010). Iran also is constantly exposed to natural disasters such as earthquakes and floods. This situation further emphasizes the importance of an Integrated system of relief logistics in place throughout the nation, because such a system benefits not only individual people but also serves the best interests of the nation.

The researches in the humanitarian logistics operation have different levels including one-level (Overstreet et al., 2011), two-level (Tzeng, et al., 2007) and multi-level (Balcik, et al., 2008). The number of facilities in this category of problems includes single-facility and multi-facilities (Jotshi, et al., 2009). The scale of transport fleet is investigated for one vehicle and several vehicles (Tzeng, et al., 2007).

Ratick et al., (2009) declare that the emergency request is probable and fuzzy in the certain humanitarian logistics operations. They investigated the duration of travel for satisfying the certain/stochastic request. Mete and Zabinsky (2010) showed that accessibility of facilities and paths is studied by two criteria: certain and stochastic. The limitation of facilities capacity in all types of models is active and sometimes passive. The emergency vehicles are classified into two types: with capacity and without capacity. The time window in considered in three categories of unlimited time (Mete and Zabinsky, 2010), soft time window (Balcick and Neamon, 2008) and hard time window (ozdamar, et al., 2004). The humanitarian logistics model can perform the delivery operation (Mete and Zabinsky, 2010), loading (Batta, 2009) and simultaneous delivery and loading (Campbell et al., 2008) in the stations. The type of path is defined in two types of
open (Jotshi, et al., 2009) and closed (Hale and Moberg, 2005). The location of customers in the network needs routing of nodes (Mete and Zabinsky, 2010; Sheu, 2007) or edges (Campbell et al., 2008). The type of objective functions is cost (Rawls et al., 2010), humanitarian (Jotshi, et al., 2009) or the combination of the two which is single-objective (Ozdamar, et al., 2004; Mete and Zabinsky, 2010), two or multiple objective (Mete and Zabinsky, 2010). These models can be employed in the crisis such as an earthquake (ozdamar, et al., 2004; Mete and Zabinsky, 2010), flood (Doerner et al., 2008) and others. The method of solving these models are precise (Balcick and Neamon, 2008), meta-heuristic (Paul and Batta, 2008) or heuristic (Doerner et al., 2008). The decision-making levels in these problems are strategic (Doerner et al., 2008), tactical (Mete and Zabinsky, 2010) or operational (Mete and Zabinsky, 2010; Ozdamar, et al., 2004). The time horizon in these issues is static (Doerner et al., 2008; Mete and Zabinsky, 2010) or dynamic (ozdamar, et al., 2004). The type of consignment in the humanitarian logistics problems can be goods (Ozdamar, et al., 2004; Mete and Zabinsky, 2010), human beings (Drezner, 2007) or a combination of these two. In some other logistics papers, the investigation is conducted based on Markov and semi-Markov chain and investigation of the air emergency considering that the problem is dynamic in order to control the fluctuations (Cheng, 2015).

Modeling of the location problems of emergency facilities has emerged in the scientific discussions during the world war two and researchers pay special attention to this field. The 1970s can be considered the years of commencement of modeling of emergency facility location. At the beginning of this decade, the primary models, LSCP and MCLP (Moeini et al., 2015), were created which became the base for the further emergency facility location models. The first model used to minimize the response time instead of objective envelopment function (Swoveland et al., 1973). Berlin and Liebman (1974) and was presented as the first dynamic model in 1974. Further, Schilling et al, (1979) represented the FLEET (Facility Location Equipment Emplacement Technique) and TEAM (Tandem Equipment Allocation Model) models with several types of servers. In the years between 1970 and 1980 most of the models were considered as certain and uncertainty was less considered in the parameters. The first probable was created in 1978 (Aly and White, 1978) in the 1980s decade most of the researchers considered the models to be probable and parameters to be random. In addition in this period the models with supportive cover (Hogan and Revelle, 1986) were most in attention. Daskin (1983) presented one of the most important probable with envelopment objective function, which provided a field for using probable models. Additionally, in this period, location-assignment (Pirkul, 1988) models were considered. ReVelle (1989) presented the first review model, which categorized the emergency facilities models. Further, Matsutomi and Ishii (1992) for the first time employed the fuzzy concept in the modeling and solving of emergency facility location problems. for the first time. Ball and Lin (1993) presented the first model considering the reliability. Marianov and Serra (1996) used the queue theory in their model. Between the years 1990 and 2000 the researchers developed the existing models. Since 2000 there has been great attention to the solving method and multiple objective models. In 2004, the partial cover model was presented by Karasakal brothers (2004), which solved the main problem of MCLP model. Erkut et al, (2009) evaluated and developed the MCLP in 4 different states. Berman et al,
Ardavan Babaei, Kamran Shahanaghi (2007) used the transfer point concept in their paper for the first time. In addition, Erkut et al. (2007) presented the significant model of MSLP with the development and troubleshooting. Also, according to the solution method caused the using of heuristic and meta-heuristic models in the papers in recent years. Rajagopalan et al. (2007) compared 4 meta-heuristic models for solving the MEXCLP problem which was presented by Daskin (1983). For example, the distribution and emergency transport issues are used in below papers. Different models in the fields of crisis based on the transport and distribution on the humanitarian logistics have been investigated which are explained in the following. In the files of transportation, the models are based on cost, time and number. The models based on the cost include the cost of travel which is comprised of maintenance and repair and distance and choosing the source with regards to the capacity and the supported area and the extent of flow under evaluation (Ben-Tal, 2011). Based on the purpose we should minimize the time of travel and the loading time with regards to the path and extent of transportation and traffic flow should be evaluated to introduce effective factors in the decision-making (Campos, 2012). In terms of shortening the distance with the request and travel time constraints and the road damages the third model is investigated (Shen et al, 2008) with regards to the fourth basis which is the number of cases that the request is not satisfied, the number of needed emergency units, the total number of covered requests, minimizing the risk and increasing the survival of people, decreasing the waiting time of injured people for help, the volume of traffic and capacity of havens based on the number of vehicle, request, the covered area, cost and time of travel has been created (Campos, 2012). The purpose of locating and assigning in the transport of humanitarian logistics is to minimize the transportation, emergency requests, the budget and resource constraints to increase the lifetime of the patients (Edrissi, 2013). As to the distribution, humanitarian logistics models concern with cost which includes minimizing travel, distribution and supply costs (Liberatore et al, 2014). With the purpose of minimizing the transportation time and the distribution time of necessary goods and the service time some papers have been researched (Vitoriano et al, 2011) Satisfying the demand and the number of emergency units needed with regards to the number of vehicles, type of vehicles and the balance flow provided a basis for the distribution of the logistics models (Vitoriano et al, 2011). Locating and assigning can be investigated based on the cost, time and number factors. With the purpose of decreasing the travel cost, the distribution cost and shortening the distance for the factor of cost a factor of time, decreasing the transportation and distribution cost, the demand factor has been investigated. The capacity of facilities is one of the main problems in humanitarian logistics. Therefore, issues such as the extent of stock and the resource flow and the number of injured are from this category (Davis, 2013). But the thing that its lack is considered the above issues is that in the beginning the stability of the problem space should be assessed in order to result in desirable and stable conclusion. From the other hand, the integrated modeling of locating the emergency stations, assigning the ambulance and the routing can lead to better and more precise results which should be considered in the presented models. This paper proposes a robust model to integrate relief supply chain which includes station locating, assigning ambulances to them and routing the ambulances for getting relief to the patients in the standard time and with the
lowest cost. To deal with uncertain nature of demand parameter (stochastic and fuzzy environment), Monte Carlo simulation method is applied to change it to the confirmed state first, and then it will behave as a stochastic and fuzzy parameter in the proposed model. Further, sensitivity analysis is applied to the main parameters to obtain more accurate results in different situations. The organization of this paper is as follows: in Section 2, we review relevant literature in emergency response planning and relief logistics. In Section 3, we present the proposed model. In Section 4, we provide numerical analysis to analyze the model. Finally, in Section 5 we provide the concluding remarks and possible future extension of this research.

**Problem statement**
Planning the ambulance is one of the important issues in the relief logistics that faces several challenges making the decision making in this field complex. The solution algorithm is presented in the Fig1.

**System dynamics and control**
Forrester (1961) considers that modeling techniques of operation research only are capable of having a limited number of variables in a system and and relations between them is linear. He employed his knowledge of feedback control and modern computers to develop methods for modeling and analysis of the problem in the complex systems. This approach focused on the dynamic behavior and systematic systems and was based on the previous researches of Tustin (1953) about the electrical and mechanical systems. Forrester continued the principles and methods of dynamic industries and implemented them in various fields. Examples of investigated dynamic systems are as follows: the process of systems engineering, biology, social, psychological and ecological systems. Forrester printed the books of principles of systems urban dynamics in (Forrester, 1975). System dynamics are a system of feedback processes which represent a determined and arranged structure. In fact, this causal structure leads to the dynamic behavior of the system. The study of complex systems is because of the great number of variables and the relation between them through the feedback loops. Forrester says The structure of a complex one feedback loop system is not simple and in this state, the system causes the determination of the behavior of the system. A complex system has many feedback loops. The rates and internal levels of this are non-linear. The complex system is of the higher order which means that the number of the variables are high. These systems usually have positive loops which represent the growth processes. In addition, it has negative or purposeful loops (Tustin, 1953). To determine the space of the critical problem, the case should be modeled and the causal diagram and the dynamic system should be used. With the causal diagram, the flow problem can be investigated by the variables. As it can be seen in Fig. 2, the problem has two rates. The input rate is proportional to the rate of services cover and the cost rate is proportional to the rate of the emergency request which means the level and number of the patients. In other words, it shows the level of the unsatisfied requests. The rate of services covered in the real world is roughly equal to the created request but is a percentage of the responsibility capacity. The responsiveness capacity is also proportional to the number of the unsatisfied patients and the minimum capacity of the emergency station and the reception capacity of the hospital. Because of the constraints in the decision-making space, the minimum capacity of the hospital and the emergency station is regarded as basic capacity.
Fig. 1. The proposed solving algorithm

Fig. 2. Flow diagram

The block which is drawn based on the causal diagram is as follows:

Fig. 3. Block diagram

Based on this the signal diagram is represented
In order to be able to identify the decisions determining and enhancing the problem space, the environment should be stable where the actual conditions are considered. To investigate the stability of a system the function of transforming the system should be found, which can be done based on the flow, blocked and signal equations. The function is found as follows

\[ H(t) = \frac{1}{\frac{1}{D} + \alpha \% \text{H}(s)} \]  

(1)

Based on the above equations the state equation is calculated as below:

\[ x(t) = x(s) \]  

(2)

\[ H(t) = H(s) \]  

(3)

The transformation function is as follows

\[ G(s) = \frac{+\alpha \% \frac{1}{S} C}{1 + \frac{1 + (s)}{S}} = \frac{\alpha \% S^{-1} C}{1 + S^{-1} H(s)} \times S^2 = \frac{+\alpha \% SC^1}{S^2 + H(s)S} \]  

(4)

\[ \Delta_1 = 1 \]  

(5)

\[ \Delta = 1 + \left[ \alpha \% H(t) \frac{1}{D} \right] \]  

(6)

\[ \text{Mason Interes} = \frac{\sum p_l \Delta}{\Delta} \]  

(7)

With regards to the below equations:

\[ \alpha_1 = H(s) \]  

(8)

\[ \alpha_2 = 0 \]  

\[ \alpha_0 = 1 \]  

(9)
Considering the stable state equation the problem is investigated.

\[ b_0 = 0 \]
\[ b_1 = \alpha \% \text{C}^1 \]
\[ b_2 = 0 \] (10)
\[ \beta_0 = 0 \]
\[ \beta_1 = \alpha \% \text{C}^1 \]
\[ \beta_2 = b_2 - \alpha_1 \beta_1 = (0 - H(s)) \alpha \% \text{C}^1 = -\alpha \% \text{CHS} \] (11)

\[ \dot{x}(t) = \begin{bmatrix} 0 \\ -H(s) \end{bmatrix} x(t) + \begin{bmatrix} +\alpha \% \\ -\alpha \% H(s) \end{bmatrix} u \] (12)

The results indicate that, both states are stable.

**Simulation**

In the Monte Carlo simulation at first a random number is determined and in the next step, the probability of the event is compared with the randomly generated number. In the state that the produced number can satisfy probability criteria, a process of a set of processes is produced. This routine repeats several times and for each repetition, a measurable output is produced. In the final section, the test set or the output results are statically analyzed and the interpretation of the results is announced. Monte Carlo simulation method is used mostly in engineering fields to predict the actual behavior and virtual systems (Nie et al, 2005). The demand is considered in this paper according to its intensity and value. Thus, in six below presented scenarios the demand is introduced based on the probability.

\[ S^2 + H(s)S = \frac{s^2}{p^2} + \frac{H(s)}{p} - 0 \] (18)

\[ 1 \text{ row } p \quad 0 \] (19)
\[ 2 \text{ row } 1 \quad 0 \] (20)

<table>
<thead>
<tr>
<th>Probability</th>
<th>0.05</th>
<th>0.15</th>
<th>0.3</th>
<th>0.25</th>
<th>0.15</th>
<th>0.1</th>
<th>( \sum p_i = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand intensity</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. The probability value and the demand intensity

Then, the uncertainty space is transformed to the certain and the results are represented in Table 2.
Table 2. The certain intensity of demand

<table>
<thead>
<tr>
<th>Random generated number</th>
<th>0.23</th>
<th>0.58</th>
<th>0.57</th>
<th>0.78</th>
<th>0.26</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity of certain demand</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

The value of the demand in the Table 3 is represented with the event probability.

Table 3. The probability and demand values

<table>
<thead>
<tr>
<th>Probability</th>
<th>0.1</th>
<th>0.15</th>
<th>0.25</th>
<th>0.2</th>
<th>0.1</th>
<th>0.1</th>
<th>0.05</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>0.4</td>
<td>35</td>
<td>30</td>
<td>25</td>
<td>20</td>
<td>0.15</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

The results of Monte Carlo simulation is presented in the Table 4.

Table 4. Confirmed values

<table>
<thead>
<tr>
<th>Random number</th>
<th>0.21</th>
<th>0.4</th>
<th>0.15</th>
<th>0.74</th>
<th>0.1</th>
<th>0.6</th>
<th>0.69</th>
<th>0.4</th>
<th>0.28</th>
<th>0.74</th>
<th>0.75</th>
<th>0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>65</td>
<td>75</td>
<td>70</td>
<td>50</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This way the demand is changing to the confirmed state. This demand is for the region one, demand for this region because in most of the situation is from morning to the evening during 5 scenarios and by coming of the night the movement is towards the region three the occupational hazards are less than 60 percent of the demands of one for three and reversely and the region two is the connecting bridge. The average of demand of one and three is determined with rounding the demands to the above.

Determining the probability of the scenario events

The scenarios based on two criteria of utility are extracted from the below model (Wang, 2016). In the presented model from three sections of the city, the two sections of first and third are in competition with each other which the game tables of them are shown based on the two criteria of traffic and population density in the below and the second section is the place of transform which is concluded from the total of the two previous sections.

The first criterion for the first row p1 and for the second row p2:

\[
\begin{bmatrix}
\frac{2}{-1} & -1 \\
1 & 1
\end{bmatrix}
\]

For the second criterion from the p3 and p4 we have:

\[
\begin{bmatrix}
\frac{1}{-1} & -1 \\
1 & 1
\end{bmatrix}
\]

Thus with using the mathematical model Ps are calculated as below:

\[
Max (p1, p2) \left[ \frac{2 + 1}{-1 - 1} - \frac{1 - 1}{1 + 2} \right] (p3) - (p1 + p2) \leq 0
\]

St.

\[
(p1, p2) \left[ \frac{2}{-1} - \frac{1}{1} \right] - (p1 + p2) \leq 0
\]

\[(22)\]
\((p_3, p_4) \begin{bmatrix} 1 \\ -1 \\ \frac{-1}{2} \end{bmatrix} - (\rho_1 + \rho_2) \leq 0\)

\[ p_1 + p_2 = 1 \]

\[ p_3 + p_4 = 1 \]

\[ p_1, p_2, p_3, p_4, \rho_1, \rho_2 \geq 0 \]

Here the \(p_1\) and \(p_2\) are the Lagrange coefficients for each criterion and \(p_5\) is proportional with the total of them. With solving the model, the below values are obtained:

**Table 5. The obtained values from the model**

<table>
<thead>
<tr>
<th>Values obtained from the model</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>1</td>
</tr>
<tr>
<td>0.4</td>
<td>2</td>
</tr>
<tr>
<td>0.4</td>
<td>3</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
</tr>
<tr>
<td>0.4</td>
<td>5</td>
</tr>
</tbody>
</table>

**Presenting the model**

The limited financial power for implementing and administration of the ambulance stations justifies the necessity of planning in this field and also because of existence of uncertain conditions in the ambulance demand at different points and time of passing the paths is different by the ambulances in various periods of the day, uncertain planning can present a more appropriate solution. Integration of relief supply chain which includes locating the stations and assigning ambulances to them and also routing of ambulances for getting relief to the patients in the standard time and with the lowest cost is the aim of this paper for designing a mathematical model. Some points in the city are the candidate for establishing and implementation of stations for maintaining ambulance and emergency helicopters in which the stations can be established with different costs. With dividing the emergency accidents in a way that the accidents with high risk and injury need emergency station and transfer to the hospital but in the situations with lower intensity, the stations, called urgent care which is introduced in this paper prevent the time waste in emergency stations services. The injured people who are able to go to the treatment center and the accident is of lower intensity are in this category we refer to the urgent cares. The beginning point of the ambulances are the emergency centers which can choose different paths in order to reach demand points and hospitals. The traffic load of these paths are time dependent and therefore a single approach cannot be chosen for all times and different scenarios are created based on the simulation. The traffic load has a direct impact on the passing time. It is possible that a long time is needed for a short path and vice versa. Therefore it is more appropriate to consider the time of passing between two points instead of the distance. Finally, it shall be decided about the selection of the destination hospital. The possibility of transferring the ambulances in order to better utilize the resources in the decision making is considered. Form one hand because of the uncertainty space in relation to the demands of the ambulances of the stations, the demands are considered based on different scenarios. The presented model has multiple steps so that in one step the assignment is made based on the scenarios of satisfying the demand points from the emergency stations. In the next step based on the final assignment the emergency locations, are established and in the last step after establishing the emergency locations the appropriate paths for them are designated. Therefore the presented model is a multi-step mode. In the below section we introduce the constituting components of the presented model.
Indices:
i: applicants
j: emergency stations
jj: candidate centers for air emergency
u: candidate centers for establishing caring-treatment
k: hospital
S: scenario

Parameters:
d_{is}: amount of demand i in the scenario S in terms of ambulance numbers
d_{is}: amount of demand i in the scenario S in terms of helicopter numbers
T: standard time
\alpha: minimum covered percentage
t_{ijk}: time from the emergency j to the demand i and transfer to the hospital k
M: big number
f_j: cost of establishing the location of emergency j
f_{jj}: cost of establishing the location of air emergency jj
g_j: cost of assigning ambulance to the emergency station j
g_{jj}: cost of assigning helicopter to the air emergency station jj
A: cost of establishing a path from i to j upon k
B: cost of establishing a special path from i to j upon k
b_\sigma: performance coefficient of assignment in each scenario
\omega: coefficient of feasible space
\lambda: coefficient of effects of scenarios
M: big number
dd: difference of fuzzy central number with lower limit
\theta: confidence percentage
d_{is}: demands average
C: cost of fuzzy assignment
\tau_{is}: track of patient entering with priority in each scenario
\theta_s: track of service in each scenario
C_{cap}: cost of establishing caring-treatment center
\text{cap}_{\sigma}: capacity of caring-treatment center
S: number of scenarios

Var(s): variance of track of patient entering with priority in each scenario

Decision making variable:
x_j: is one if the location j is established as the emergency station otherwise it is zero.
x_{jj}: is one if the location jj is established as the air emergency station otherwise it is zero.
z_{jj}: the extent of assigned helicopter to the air emergency station jj
y_{is}: the extent of needed ambulance of demand i in the station j under scenario S
y_{is}: the extent of needed ambulance or helicopter of demand i in the station j or jj under scenario S
y_{is}: the extent of needed helicopter of demand I in the station jj under scenario S
z_j: the extent of assigned ambulance to the emergency station j
x_{ik}: is one if the path of emergency j to the demand i upon the hospital k is established otherwise is zero.
x_{ik}: is one if the special path of emergency j to the demand i upon the hospital k is established otherwise is zero.
P and V, converter variable
Q_{is}: number of service ambulance to the demands in each scenario
U_{cap}: if the caring-treatment center is established is one otherwise is zero.

Robust model
To provide a model for final assignment of the ambulance based on the extent of needed ambulances at each demand point, the deviation from the feasible space is being considered. The extent of impact of one scenario on another is considered in the objective function which means that in each scenario the extent of the needed ambulance at each demand point causes difference in another scenario (Mulvey, 1995).
\[ M \ln \left( \sum_{j} (f_{j}x_{j} + g_{j}z_{j}) \right) \]
\[ + \sum_{i} \sum_{s} \sum_{f} \left( Ax_{i,j,k} + B x_{i,j,k} \right) \]
\[ + \sum_{s} \sum_{i} \sum_{f} p_{s} y_{i,s} \]
\[ + \lambda \sum_{s} p_{s} \left( \sum_{i} \sum_{j} y_{i,s} - \sum_{s} p_{s} \sum_{i} \sum_{j} y_{i,s} + 2\theta_{s} \right) \]
\[ + w \sum_{s} p_{s} (\delta_{s} + 2\Delta_{s}) \]
\[ + \left( \frac{r_{s}}{\varphi_{s}^d} \right)^{Q_{is}-1} \left( \frac{r_{s}^d}{\varphi_{s}^d} \right)^{d} \sum_{d} \frac{1}{d!} \left( \frac{r_{s}}{\varphi_{s}^d} \right)^{Q_{is}} \frac{1}{(Q_{is} - 1)!} \left( \frac{r_{s}}{\varphi_{s}^d} \right)^{Q_{is}} \right) \]
\[ + \sum_{i} \sum_{s} \frac{b_{s} \gamma_{i,s}}{d_{i,s} + d_{dd_{i,s}} - b_{s} \gamma_{i,s}} \]
\[ + \sum_{u} \sigma_{u} U_{C_{u}} \]

S.t.

\[ -Mx_{j} + z_{j} \leq 0 \quad \forall j \]  
\[ \sum_{i} y_{i,s} - b_{s} \gamma_{i,s} + \delta_{s} = 0 \quad \forall j \]  
\[ \alpha \sum_{i} \sum_{s} d_{i,s} - \sum_{i} \sum_{s} \sum_{j} y_{i,s} \leq 0 \]
\[ \beta \sum_{i} \sum_{s} d_{dd_{i,s}} - \sum_{i} \sum_{s} \sum_{j} y_{i,s} \leq 0 \]
\[ (1 - \alpha) \sum_{i} \sum_{s} d_{i,s} + (1 - \beta) \sum_{i} \sum_{s} d_{dd_{i,s}} + (1 - \alpha) \sum_{i} \sum_{s} \sum_{dd_{i,s}} \leq S \sum_{u} \sigma_{u} U_{C_{u}} \]

\[ \sum_{i} y_{i,s} - d_{i,s} \leq 0 \quad \forall i \]  
\[ \tau_{i,j,k} x_{i,j,k} - M x_{i,j,k} - T \leq 0 \quad \forall i,j,k \]
\[ x_{j} - \sum_{k} x_{i,j,k} \leq 0 \quad \forall j \]
\[ -\theta_{s} - \sum_{j} y_{i,s} + \sum_{s} p_{s} \sum_{i} \sum_{j} y_{i,s} \leq 0 \quad \forall s \]
\[ -\Delta_{s} - \delta_{s} \leq 0 \quad \forall s \]
\[ \sum_{k} x_{i,j,k} \leq M \sigma_{j} \quad \forall j \]
The objective function (Equation 1) has nine components. The first component is minimizes the cost of establishing the emergency location and the cost of final assignment of the ambulance to the emergency station. The second component intends to minimize the cost of establishing the path and the special path. The third component intends to minimizes the number of needed ambulances in each scenario. The fourth and fifth components are the main parts of model. The sixth component intend to minimize the waiting time of the applicants. SO that the entering of the applicants and patients are based on the Poisson process and the service time of the random variable is arbitrary and the system with $Q_{u_{ts}}$ is the server. This queue is according to the M/G/m queue pattern (Sundarapandian, 2009) The seventh component intends to minimize the cost of establishing air emergency station and emergency helicopter. The eighth component intends to increase the rate of the response to the demands in the emergency. The ninth component intends to decrease the establishment cost of urgent care. When the ambulance is assigned to the station location, a location should be established there (Equation 2). The final assignment to one location is based on the number of the ambulances that is demanded in each scenario (Equation 3). The total ambulances that are needed in each scenario in each place should be at least $\mu$ percent of total demand (Equation 4). The sum of emergency helicopters in each scenario should at least be the $\beta$ percent of total demands (Equation 4a). The sum of demands is lower than the responded emergency level (Equation 4b). The number of needed ambulances in each scenario is lower than the demand value (Equation 5). The sum of time in a path should be lower than the standard time $T$ otherwise the path is transformed into a special path in other words when the sum of transportation time from the emergency station to the demand location and to the hospital is higher than the standard time designated by the experts the path should decrease the time in terms of management and physical structure. In terms of management based on the traffic control of the region and in terms of structure by establishing special path. The path is constructed when the emergency station is established (Equation 6). The path is constructed by the number of assigned ambulances (Equation 7). Equation 8 and Equation 9 are linearization limitations of robust. Equation 10 shows the guarantee of constructing the path that the station has the ambulance. Equation 11 shows the number of services. When the assignment of the helicopter to the station is done it shall be established a place as the
emergency air station (Equation 12). The final assignment to a location is based on the number of the emergency helicopter that is demanded from that location in each scenario (Equation 13). The sum of emergency helicopters that are needed in each scenario in each location shall be at least $α$ of the total demands (Equation 14). Equation 15 specify the type of model variables.

**Hybrid robust model**

In this model, the extent of the assigned cost to the ambulance in each demand point proportional to the location of the emergency stations is stated in a triangular fuzzy manner. Therefore the model intends to increase the response chance for the needs of each demand point and in addition in this model, the extent of the assigned cost of the ambulance to each demand point proportional to the location of the emergency station is stated as fuzzy triangular. Hence the model tries to increase the response chance to the needs of each demand point (Wang, 2009).

In this model, the changes are made in the Equation 5 and proportional to the type of the problem two equations of 16 and 17 are added. The number of needed ambulances for each demand in each scenario in less than the demand value (Equation 5a). The chance that the demand is a percentage of the demand point in each scenario from each location is based on the probability distribution. The distribution for the probability coefficients of the variable is based on the normal distribution with average zero and variance of one (Equation 5b). The chance that the demand is a percentage of the demand point in each scenario of each location is based on the fuzzy demand. For this limitation, the assumption is the maximum of 50 percent confidence (Equation 5c). For solving the problem, the minimum of the Equation 5b and Equation 5c are considered. Because the solution to both the problems is feasible but it should be selected based on the conditions of the problem.

**Analysis of the results**

With regards to the fact that the hessian matrix is the limitations of the feasible region and the objective function of the positive half and the problem is defined linear therefore the feasible space of the problem and the objective function are convex. Hence in this type of convex planning, the problem has optimum point which is attained by solving the problem and in simper words, the solution is convergent to the optimum point (Fiacco and Mccormick, 1968). By considering the four-candidate location of the emergency stations for satisfying the three demand points for getting the injured to the three hospitals are being analyzed. The problem 1 with the limitation of assignment scenario is based on the fuzzy demand of the demands and the problem 2 is assignment scenario based on the probable satisfaction of demand. Parameters $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, $\bar{\delta}$ existing in the problem in the interval of 20 percent decrease to 20 percent increase are changing in order to investigate the extent of sensitivity of the problem towards them. In the tables and diagrams below the impact of them on the objective function of the effective problem and main variables of the problem are investigated. The model is analyzed by GAMS and Baron Solver software in 3600 seconds with linearization and simplification.
\begin{equation}
M \sum_{i} (x_i \gamma_i + \eta_i \eta_i) \\
+ \sum_{j} \sum_{l} \left( \alpha \xi_{ijkl} + \beta \xi_{ijkl} \right) \\
+ \sum_{j} \sum_{l} \eta_{il} \eta_{lj} \\
+ \sum_{j} \sum_{l} p_{ij} \left( \sum_{k} \eta_{ik} + \sum_{k} \eta_{lk} + 2 \eta_{ij} \right) + \sum_{j} \sum_{l} p_{ij} (\sigma_j + 2 \delta_j) \\
+ \sum_{j} \sum_{l} \left( \frac{1}{2} (\eta_{ij}^{\text{max}} - \eta_{ij}^{\text{min}}) \right) \left( \frac{1}{2} (\eta_{lj}^{\text{max}} - \eta_{lj}^{\text{min}}) \right) \\
+ \sum_{j} \sum_{l} (\eta_{ij} \eta_{lj} + \eta_{ij} \eta_{lj}) - \sum_{j} \sum_{l} \left( \frac{b_{ij} \delta_j + b_{lj} \delta_l}{\tau_{ij} \tau_{lj}} \right)
\end{equation}

S.t.
\begin{align}
-M \chi_j + z_j & \leq 0 \quad \forall j \\
\sum_{l} \eta_{lj} + \sum_{s} \eta_{ls} & = 0 \quad \forall j, s \\
\alpha & \sum_{l} \sum_{j} d_{ls} - \sum_{l} \sum_{j} \eta_{lj} \leq 0 \\
\beta & \sum_{l} \sum_{j} d_{ls} - \sum_{l} \sum_{j} \eta_{lj} \leq 0 \\
(1 - \alpha) & \sum_{l} \sum_{j} d_{ls} + (1 - \beta) & \sum_{l} \sum_{j} d_{ls} + (1 - \alpha) & \sum_{l} \sum_{j} d_{ls} \leq S \sum_{l} \cap_{ul} U C_{ul} \\
\sum_{l} \eta_{lj} & \leq d_{ls} \leq 0 \quad \forall l, s \\
\phi_{ij}^{-1} \eta_{lj} & \leq d_{ls} - d_{ls} \quad \forall l, j, s \\
\eta_{lj} & \leq (1 - 2 \theta)(d_{ls} - d_{ls}) + 2 \theta d_{ls} \quad \forall l, j, s \\
\tau_{ij} \chi_{ijk} - M \chi_{ijk} - T & \leq 0 \quad \forall l, j, k \\
\chi_j - \sum_{l} \sum_{k} x_{ijk} & \leq 0 \quad \forall j \\
\eta_{lj} - M \sum_{k} x_{ijk} & \leq 0 \quad \forall l, j \\
-\delta_j - \sum_{l} \sum_{j} \eta_{lj} + \sum_{l} p_{ij} \sum_{j} \sum_{l} \eta_{lj} & \leq 0 \quad \forall j \\
-\Delta_{ij} - \delta_{ij} & \leq 0 \quad \forall j \\
\sum_{l} \sum_{j} \sum_{s} \eta_{lj} & = V \\
\sum_{l} \sum_{j} \sum_{s} E(d) p_{ij} \eta_{lj} & \leq V \\
\sum_{l} \sum_{j} x_{ijk} & \leq M \chi_j \quad \forall j
\end{align}
Table 6. changes of $\omega$

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Solving the problem for changes of $\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td><strong>Problem cost 1</strong></td>
<td>849.1511</td>
</tr>
<tr>
<td><strong>Problem cost 2</strong></td>
<td>964.3009</td>
</tr>
</tbody>
</table>

As it can be seen in the Fig. 5 for increasing $\omega$ the cost increases and decreases. For 10 percent decrease, the cost parameter of the problem 1 decreases and for 10 percent more decrease from the point 0.9 it decreases with almost the same slope. For 10 percent increase, the cost parameter decreases remarkably and make 10 percent more increase the cost of problem 1 increases. With the increase of the number of ambulances assigned to each emergency station the cost increases until that the response chance to the demands of the objective function decreases. On the other hand with the increase of the cost assigned to the
ambulances in the objective function the cost increases. In the minimum point, we observe a system with high response chance and low objective function value. As it can be seen in the Fig. 6 in the problem 2 for the increase of the total cost increases. These changes are constant except in one point. For 10 percent increase from the point 1.1 in the problem 2 the cost increases with the lower slope.

With the increase of the feasible space, the number of assigned ambulances in each emergency station increases. In this situation, because more demand shall be met the increase in the number of the ambulances leads to increase of the cost. In general, the cost of this problem is more than the previous problem but more demand is covered.

Fig. 6. changes in the second problem for changes of \( \omega \)

Table 7. changes of \( \lambda \)

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Solving the problem for changes of ( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td><strong>Problem cost 1</strong></td>
<td>847.9331</td>
</tr>
<tr>
<td><strong>Problem cost 2</strong></td>
<td>967.4988</td>
</tr>
</tbody>
</table>

Fig. 7. changes of objective in problem 1 for changes of \( \lambda \)
As it can be seen in the Fig. 7 in the problem 1 for the increase of $\lambda$ the total cost increases except in one point. For the increase of 10 percent, the cost parameter of the problem 1 decreases, then for 10 percent more decrease, the cost of the problem 1 decreases with the same slope. For 10 percent decrease of the parameter the cost of problem 1 increases. Then for 10 percent more decrease the cost decreases remarkably. If a good assignment leads to the responsiveness of the scenarios the more ambulances are assigned which results in the increase of the costs and this trend continues until the decrease of the function value in order to maximize the change.

As it can be seen in the Fig. 8 in the problem 2 like the problem 1 for the increase of $\lambda$ the total cost decreases and increases. For 10 percent increase the cost parameter of problem 2 decreases. Then for 10 percent more increase, the cost of problem 2 decreases with the lower slope. For 10 percent decrease the cost parameter of the problem 2 decreases and for 10 percent more decrease the cost of problem 2 increases. The increase of the cost in face of assigning more ambulances leads to increase of the costs and create more change. The response chance to the demands continues until it reaches its maximum limit and on one hand in this problem, more demand is met therefore with increasing the response change, more control is created in the problem and assignment is in a way that most scenarios are satisfied well.

![Fig. 8. changes of objective in problem two for changes of $\lambda$](image)

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Solving the problem for changes of $\varnothing^{-1}(\alpha)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem cost 2</strong></td>
<td>852.0806</td>
</tr>
</tbody>
</table>

Table 10. changes of $\varnothing^{-1}(\alpha)$
As it can be seen in the problem 2 for increase of the parameter $\phi_{(k)}$, the total cost decreases except in one point. The changes of the objective are remarkable in the begin and end interval. For 10 percent increase the cost parameter of the problem 2 decreases, for 10 percent increase also the total cost decreases. For 10 percent decrease the cost of problem 2 increases merely and then for 10 percent more
decrease the problem cost 2 decreases. With increasing the probability of ambulance efficiency, the number of assigned ambulances decreases and fist the chance of satisfying the demand decreases, therefore, the value of objective function increases but this continues until the cost of non-assignment and even not the establishment of stations increase the cost.
As it can be seen in the Fig. 10 in the problem 1 for the increase of parameter the cost of problem 2 decreases and increase and for the increase of the parameter the cost of problem 2 increases merely in all intervals. Then for 10 percent decrease the parameter of cost in problem 2 decreases then for 10 percent more decrease the cost of problem 2 decreases significantly. With the decrease of the demand for land ambulance a fixation of demand for land and air demand the chance of objective function decreases which leads to increase until it reaches constant items and it shall be established dependent on the ambulance demand.

As it can be seen in the Fig. 11 in problem 1 for increasing the parameter the total cost decreases and increases. For 10 percent increase of the parameter the problem cost decreases. Then for extra 10 percent increase, the problem cost 1 increases. For 10 percent decrease of the parameter the problem cost 1 decreases dramatically. Then for 10 percent decrease from the point 0.9, the problem cost decreases insignificantly. With increasing contact density and number of demands the responsiveness increases until the queue is formed and this increase continues until the system is not able to response and the demands are lost therefore the costs increase and the value of objective function increase.

As it can be seen in the Fig. 12 in the problem 2 for the increase of the parameter. The total cost decreases, and increases. For 10 percent increase of the parameter the cost of the problem 2 decreases. Then for 10 percent more increase, the problem cost 2 increases. For 10 percent decrease of the parameter the problem cost 2 decreases significantly. Then for 10 percent more decrease from the point 0.9 the problem cost 1 decreases insignificantly. With the increase of calls in a certain time the queue is formed until it cannot be satisfying and finally the demand is lost therefore more cost is created and the objective function increases.

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Solving the problem for changes of $\tau_{og}$</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
<th>1.1</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem cost 1</td>
<td>841.9092</td>
<td>841.9107</td>
<td>851.0688</td>
<td>850.5026</td>
<td>853.9222</td>
<td></td>
</tr>
<tr>
<td>Problem cost 2</td>
<td>955.8696</td>
<td>955.9921</td>
<td>966.2244</td>
<td>965.7822</td>
<td>969.1101</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 11. changes of objective in first problem for changes of $\tau_{og}$
Fig. 12. changes of objective in the second problem for changes of $r_{DF}$

Table 12. changes of $\varphi_3$

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Solving the problem for changes of $\varphi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td>Problem cost 1</td>
<td>967.4823</td>
</tr>
<tr>
<td>Problem cost 2</td>
<td>823.9452</td>
</tr>
</tbody>
</table>

Fig. 13. changes of objective in the first problem for changes of $\varphi_3$

As it can be observed in Fig. 13, in the problem 1 for increasing the parameter $\varphi_3$, the total cost changes. For 10 percent decrease, the problem cost 1 increases. Then for 10 percent more decrease from the point 0.9 the problem cost 1 increases remarkably. For 10 percent increase of the parameter the problem cost 1 increases. Then for 10 percent more increase, the problem cost one increases and for 10 percent more increase the problem cost decreases. With increasing the service rate, the queue length decreases and the system works with more efficiency but the efficiency of each ambulance increases and more assigned ambulances are
substitutes and the change of meeting the demand decreases and the increases of the objective function continues until the decrease of the cost for assignment and decrease of queue length decreases the objective function.

As it can be seen in the above diagram in the problem 2 for increasing of parameter $\theta_s$, the total cost shows a symmetric behavior. For the decrease of 10 percent, the cost of the problem 2 decreases dramatically. Then for 10 percent more decrease from the point 0.9 the cost of problem 1 increases. For 10 percent increase, the problem cost 2 decreases then for 10 percent increase the problem cost decreases. With increasing the service rate the ambulance efficiency increases, therefore, the assignment cost decreases.

In this problem, the demand is more satisfied and the decrease of assignment leads to the decrease of satisfied chance and therefore problem intends to satisfy more until the changes reach its peak and the decreased assignment leads to less establishment and assignment chance and with the increase of the service rate the objective function decreases finally. Based on the findings it can be said that in the problem two more demand is investigated and satisfied. In general for finding the balance between the change of satisfying the demand and reducing the costs the problems shall be considered. In the analyzed diagrams the consequence of constant slope is increasing and decreasing. In general for analyzing the problem and employing presented models the problem space shall be fully known. Then for deciding the proper analysis according to the condition shall be used.

Sometimes we face conditions that shall make decisions in the shortest time to get most results, therefore, it can be decided on the increasing slopes for decision making while in some cases the environmental limitations prevent such decisions so the decreasing slopes shall be used. Sometimes the extent of investment in the special case does not affect the results therefore with above analysis can prevent such wastes.

**Fig. 14. changes of objective in the second problem for changes of $\theta_s$.**
Conclusion
Relief in times of accident has special importance because of creating risk for their lives. For decreasing the risk and increasing the survival of the injured the relief should be in a standard time. On the other hand for creating an integrative relief system there are limitations such as financial resources. On the other hand for better facing with the relief problem, this issue should be investigated in the uncertainty space in order to make the results closer to the reality. In this paper, the critical problem and relief logistics are investigated through eight steps. In the first step with using the system dynamics, the relief problem space is drawn. In the second step, the causal space from the viewpoint of the control is investigated. In the third step by using the simulation the demand to the ambulances is attained. In the fourth step the probability of the occurrence of scenario events with regards to the game theory and according to the traffic and population density is calculated. In the fifth step integrated multi-step modeling of locating the emergency stations, assigning and routing of ambulances with regards to the satisfaction of fuzzy and probable demand and according to minimizing the fuzzy cost based on each scenario is conducted using robust planning. In the sixth step, the problem is analyzed from the viewpoint of the results. In the results of this problem, it has been indicated that the model with probable limitations, leads to more demand satisfaction and both problems intend to generate the solution with regards to the balance between the change of demand satisfaction and reducing the costs that these problems are investigated in the whole system. In general, based on the presented model, the problem shall be identified first and decisions shall be made stepwise in relation to the conditions and limitations.

For future studies the suggestions are made as follows:
• Locating the emergency centers in moving manner in various periods should be investigated.
• Ambulances should be leveled according to the presented services.
• Ambulances with support service should be assigned to the problem. With analysis of the sensitivity of the number of support ambulances in each emergency location with regards to the requested demand is determined to increase the confidence coefficient.
• The distribution of employing the ambulances should be determined so that the emergency locations can lend their idle ambulances to the locations with higher demand.
• Maintaining and repairing the ambulances and paths should be investigated so that the extent of access to these resources is determined.
• For more distribution of equality, an objective should be defined as the decrease of the unsatisfied demand.

Reference
ambulance location. Socio Economic Planning Sciences, 8(6), 323-328.
47. Swiss, R. Natural catastrophes and man-made disasters in 2014: convective and winter storms generate most losses. Sigma, (2)